



Personal Finance: Another Perspective

Time Value of Money: A Self-test

Updated 2017-01-20



Objectives

- A. Understand the importance compound interest and time
- B. Pass an un-graded assessment test with 9 problems



How Important is Interest?

Albert Einstein stated: “Compound interest is the eighth wonder of the world”

Following are seven “Time Value of Money” problems to test your knowledge. You should already know how to do these types of problems.



Assessment #1: Pay or Earn Interest

- It is estimated that most individuals pay \$1,200 per year in interest costs. Assuming you are 25 and instead of paying interest, you “decide to decide” to earn it. You do not go into debt, but instead invest that \$1,200 per year that you would have paid in interest in an equity mutual fund that earns an 8% return. How much money would you have in that fund at age 50 (25 years) assuming payments are at the end of each year and it is in a Roth account in which you pay no additional taxes? At age 75 (50 years)?



Answer #1: Interest

- Clear your registers (memory) first
- Payment = \$1,200 Payment = \$1,200
- Years (n) = 25 Years (N) = 50
 Interest rate (I) = 8%
- Future Value at 50 = \$87,727
- Future Value at 75 = \$688,524

**Not a bad payoff for just not
going into debt!**



Assessment #2: The Savings Model

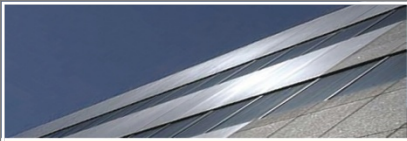
- Suppose you have \$2,000 per year to invest in a Roth IRA at the beginning of each year in which you will pay no taxes when you take it out after age 59½. What will be your future value after 40 years if you assume:
 - A. 0% interest?
 - B. 8% interest (but only on your invested amount)?, and
 - C. 8% interest on both principal and interest?
- What was the difference between:
 - D. $B - A$? $C - A$? $C - B$?



Answer #2: Savings

- A. Earnings at 0% interest
 - $2,000 * 40 \text{ years} = \$80,000$
- B. Earnings with 8% only on Principal
 - Total Number of periods of interest (note that the first \$2,000 has 40 years of interest, the next \$2,000 has 39 years, etc., $(40+39+38\dots+1) = 820$ periods times interest earned of \$160 (or $8\% * 2,000$) + \$80,000 principal ($40 \text{ years} * \$2,000$) = \$211,200
- C. Total earnings with principal and interest
 - Beginning of Year mode: $40=N$ $I=8$ $-2,000 = \text{PMT}$
 $\text{FV}=\$559,562$
- Difference
 - $B-A = \$131,200$ $C-A = \$479,562$ $C-B = \$348,362$

What a difference compounding makes!!!



Answer #2

Growth Strategies for Investments				
Payments are at the beginning of the Year				
	Years to	0% Interest	8% Interest	8% Interest
Year #	Invest		on Principal	on Prin & Interest
1	40	2,000	8,400	\$43,449
2	39	2,000	8,240	\$40,231
3	38	2,000	8,080	\$37,251
4	37	2,000	7,920	\$34,491
5	36	2,000	7,760	\$31,936
6	35	2,000	7,600	\$29,571
7	34	2,000	7,440	\$27,380
8	33	2,000	7,280	\$25,352
9	32	2,000	7,120	\$23,474
10	31	2,000	6,960	\$21,735
11	30	2,000	6,800	\$20,125
12	29	2,000	6,640	\$18,635
13	28	2,000	6,480	\$17,254
14	27	2,000	6,320	\$15,976
15	26	2,000	6,160	\$14,793
16	25	2,000	6,000	\$13,697
17	24	2,000	5,840	\$12,682
18	23	2,000	5,680	\$11,743
19	22	2,000	5,520	\$10,873
20	21	2,000	5,360	\$10,068



Answer #2 (continued)

21	20	2,000	5,200	\$9,322
22	19	2,000	5,040	\$8,631
23	18	2,000	4,880	\$7,992
24	17	2,000	4,720	\$7,400
25	16	2,000	4,560	\$6,852
26	15	2,000	4,400	\$6,344
27	14	2,000	4,240	\$5,874
28	13	2,000	4,080	\$5,439
29	12	2,000	3,920	\$5,036
30	11	2,000	3,760	\$4,663
31	10	2,000	3,600	\$4,318
32	9	2,000	3,440	\$3,998
33	8	2,000	3,280	\$3,702
34	7	2,000	3,120	\$3,428
35	6	2,000	2,960	\$3,174
36	5	2,000	2,800	\$2,939
37	4	2,000	2,640	\$2,721
38	3	2,000	2,480	\$2,519
39	2	2,000	2,320	\$2,333
40	1	2,000	2,160	\$2,160

\$80,000

\$211,200

\$559,562

Net impact of interest on interest

\$348,362



Assessment #3: The Expensive Car

- You graduate from BYU and you really want that new \$35,000 BMW 320i that your buddy has. You estimate that you can borrow the money for the car at 9%, paying \$8,718 per year for 5 years.
 - (a) You buy the car now and begin investing in year 6 the \$8,718 per year for 25 years at 9%.
 - (b) You keep your old Honda Civic with 150,000 miles and invest the \$8,718 per year for the full 30 years at 9%.
- Even though 9% may be a high return to obtain, what is the difference in future value between thought (a) and thought (b)? What was the cost of the car in retirement terms?



Answer #3: The Car

- Payment = \$8,718, $N = 25$, $I = 9\%$
 - Future value = \$738,422
- Payment = \$8,718, $N = 30$, $I = 9\%$
 - Future value = \$1,188,329
 - The cost of the car in retirement terms is
\$449,907

That is one expensive beamer!



Assessment #4: The Costly Mistake

- Bob and Bill are both currently 45 years old. Both are concerned for retirement; however, Bob begins investing now with \$4,000 per year at the end of each year for 10 years, but then doesn't invest for 10 years. Bill, on the other hand, doesn't invest for 10 years, but then invests the same \$4,000 per year for 10 years. Assuming a 9% return, who will have the highest amount saved when they both turn 65?



Answer #4: The Costly Mistake

Time makes a real difference (10% return)

Age:	Bob	Tom
46		4,000
47		4,000
48		4,000
49		4,000
50		4,000
51		4,000
52		4,000
53		4,000
54		4,000
55		4,000
56	4,000	
57	4,000	
58	4,000	
59	4,000	
60	4,000	
61	4,000	
62	4,000	
63	4,000	
64	4,000	
65	4,000	
	\$63,750	\$165,350

Time Really makes a difference!



Answer #4: The Costly Mistake (continued)

Clear memories, set calculator to end mode.

Solve for Bill:

$N = 10$ $PMT = -4,000$ $I = 9\%$, solve for FV

$FV = \$60,771$

Solve for Bob:

1. $N = 10$ $PMT = -4,000$ $I = 9\%$, solve for FV

$FV = \$60,771$

2. $N = 10$ $PV = 60,771$ $I = 9\%$, solve for FV

$FV = \$143,867$

**Bob will have \$83,096 more than Bill –
Begin Investing Now!!**



Assessment #5: Credit Cards

- Your friend just got married and had to have a new living room set from the Furniture Barn down the street. It was a nice set that cost him \$3,000. They said he only had to pay \$60 per month—only \$2 per day.
 - ✓ a. At the stated interest rate of 24.99%, how long will it take your friend to pay off the living room set?
 - ✓ b. How much will your friend pay each month to pay it off in 30 years?
 - ✓ c. Why do companies have such a low minimum payoff amount each month?



Answer #5: Credit Cards

- a. Given an interest rate of 24.99% and a \$3,000 loan, your friend will be paying for this furniture set for the rest of his life. He will never pay it off.
- Clear memory, set payments to end mode, set payments to 12 (monthly) $I = 24.99$ $PV = -\$3,000$, and solve for N . Your answer should be no solution.
- c. How much would your friend have to pay each month to pay off the loan in 30 years? First, do you think your living room set will last that long?
- Clear memory, set payments to end mode, set payments to 12 (monthly) $I = 24.99$ $PV = -\$3,000$, $N = 360$ and solve for PMT . His payment would be \$62.51.



Answer #5: Credit Cards (continued)

- C. Why do companies have such a low minimum payoff amount each month?
- So they can earn lots of your money from fees and interest!
 - This is money you shouldn't be paying them—Earn interest, don't pay interest!

Minimum payments are not to be nice, but to keep you paying them interest for as long as they can!



Assessment #6: Adjusting for Inflation

- Assuming you have an investment making a 30% return, and inflation of 20%, what is your real return on this investment?



Answer #6: Inflation

The traditional (and incorrect) method for calculating real returns is: Nominal return – inflation = real return. This would give:

$$30\% - 20\% = 10\%$$

The correct method is:

$$(1 + \text{nominal return}) / (1 + \text{inflation}) - 1 = \text{real return}$$

$$(1.30 / 1.20) - 1 = 8.33\%$$

The traditional method overstates return in this example by 20% (10%/8.33%)

Be very careful of inflation, especially high inflation!!



Answer #6: Inflation (continued)

- While some have argued that it is OK to subtract inflation (π) from your nominal return (r_{nom}), this overstates your real return (r_{real}).

The linking formula is:

$$(1+r_{\text{real}}) * (1+\pi) = (1 + r_{\text{nom}})$$

Multiplied out and simplified:

$$r_{\text{real}} + \pi + [r_{\text{real}} \pi] = r_{\text{nom}}$$

Assuming the cross term $[r_{\text{real}} \pi]$ is small, the formula condenses to:

$$r_{\text{real}} + \pi = r_{\text{nom}} \text{ or the Fisher Equation}$$

- The correct method is to divide both sides by $(1+\pi)$ and subtract 1 to give:

$$r_{\text{real}} = [(1 + r_{\text{nom}}) / (1+\pi)] - 1$$



Assessment #7: Effective Interest Rates

Which investment would you rather own and why?

<u>Investment</u>	<u>Return</u>	<u>Compounding</u>
Investment A	12.0%	annually
Investment B	11.9%	semi-annually
Investment C	11.8%	quarterly
Investment D	11.7%	daily



Answer #7: Effective Interest Rates

The formula is $((1 + \text{return}/\text{period})^{\text{period}}) - 1$

- 12.0% compounded annually:
 - $(1 + .12/1)^1 - 1 = 12.00\%$
- 11.9% compounded biannually:
 - $(1 + .119/2)^2 - 1 = 12.25\%$
- 11.8% compounded quarterly:
 - $(1 + .118/4)^4 - 1 = 12.33\%$
- 11.7% compounded daily (assume a 365 day year):
 - $(1 + .117/365)^{365} - 1 = 12.41\%$
- Even though D has a lower annual return, due to the compounding, it has a higher effective interest rate.

How you compound makes a difference!



Assessment #8: Earning \$1 to Spend

- Emilee has been thinking about how much she has to earn to spend \$1 once she leaves school. Assume she will be in the 25% Federal marginal tax bracket after school, living in New York (10% state marginal tax rate) and New York City (5% city marginal tax rate), and she pays 12% gross of her income to charity.

Calculations

- A. How much must she earn in New York City to spend \$1.00?
- B. How much does this increase if she has a goal to pay herself 20%?



Assessment #8 Answer

Calculations

- A. To earn \$1.00 to spend living in New York City requires:
 - Federal tax rate: 25%
 - State tax rate: 10%
 - City tax rate: 5%
 - Charitable contributions: 12%
 - The formula is:
 - $x - .25x - .10x - .05x - .12x = 1$. Solve for x?
 - $X = (1/(1 - (.25+.10+.05+.12)))$ or 2.08
 - **Emilee must earn \$2.08 to be able to spend \$1.00.**
 - (see Learning Tool TT35 Loan Amount to Pay Back)
- B. If she pays herself 20% in NYC, she must earn \$3.57



Assessment #9: Total to Pay Back

- Use the tax and charity information from the previous question. Emilee is in her second to last year in school (24 months till graduation) and is considering a \$5,000 alternative loan at 12% and plans to pay it back in 60 months after she graduates.

Calculations

- A. How much must she earn to pay back that alternative loan of \$5,000 (which is not subsidized and accrues interest while she is in school) at 12% interest over 60 months including taxes charitable contributions?



Assessment #9 Answer

Principle and interest calculations (it is easier to use LT34)

- To pay back \$5,000 in student loans requires:
 - At 12% interest and in her second to last year of school, she will add 24 months of interest or \$1,349
 - $PV = \$5,000$, rate = 12%, periods = 24, $FV = ?$
 - Future value = \$6,349
 - To pay off \$6,349 for 60 months will require:
 - $PV = 5,000$, Rate = 12%/12, Periods = 60, $PMT = ?$
 - Payment = \$141.22
 - Her total payments will be $\$141.22 * 60$ months
 - **Total Payments = \$8,474 or 69% more than borrowed**



Assessment #9 Answer

Calculations

- To determine how much she needs to earn to pay back this \$8,474, we determine:
 - Taxes:
 - Federal tax rate: 25%
 - State tax rate: 10%
 - City tax rate: 5%
 - Charitable contributions: 12%
 - The formula is $x - .25x - .10x - .05x - .12x = 1$ or $X = 2.08$
 - To pay back this \$8,474, Emilee must earn 2.08 * \$8,474 or \$17,653
- **Emilee must earn \$3.53 for every \$1.00 she borrows! Avoid debt!** ($\$17,653 / \$5,000 = \$3.53$)



Assessment Review

- How did you do?
 - If you missed any problems, go back and understand why you missed them. This foundation is critical for the remainder of the work we will be doing in class.



Review of Objectives

- A. Do you understand the importance compound interest and time?
- B. Did you pass the un-graded assessment test?